Mathematical Analysis- Exams

What is this sheet?

This sheet contains the most important formulas, definitions, strategies etc. you have to know for Exam 1 of MTH 1110- Mathematical Analysis. It does not contain any examples and it does not tell you how to use these formulas and definitions. The latter you will learn by practicing the homework problems again and making sure you know how to solve all of them.

If you have any questions, please ask.

Formulas

1.1 Graphs of Equations

1. Distance Formula: The distance from point (x_1, y_1) to point (x_2, y_2) equals $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.
2. Equation of a Circle with radius r and center (h, k): $(x-h)^2+(y-k)^2 = r^2$.

1.2 Linear Equations in One Variable

Strategy for solving linear equations:

1. Expand, if needed.
2. Separate all terms that contain the unknown (x) on the one side and all other terms on the other side.
3. Combine like terms.
4. Divide by the coefficient of the unknown (x).

1.3 Modeling with Linear Equations

Strategy for solving an equation with many variables, say for the variable a.

1. Expand, if needed.
2. Separate all terms that contain the variable a on the one side and all other terms on the other side.
3. From all the terms that contain a, factor a out.
4. Divide by the expression in front of the variable a.

Steps for solving a word problem:

1. Identify unknown and unknown quantities.

2. Assign variables to unknown quantities.
3. Identify relationships between the known and unknown quantities.
4. Express these relationships as equations.
5. Solve the equations to find the unknowns.

1.4 Quadratic Equations and Applications

1. Quadratic Formula for solving $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

   The quantity $b^2 - 4ac$ under the square root is called the *discriminant*. If the discriminant is positive, the equation has two roots; if the discriminant is zero, the equation has one root; if the discriminant is negative, the equation has no roots.

2. *Completing the square* refers to process of writing a quadratic equation $ax^2 + bx + c$ in the form $a(x-h)^2 + k$, where $(x-h)^2$ is a complete square and a, k are numbers.

   Case I: $a = 1$, i.e. complete the square of $x^2 + bx + c$. Add and subtract $(\frac{b}{2})^2$:

$$x^2 + bx + c = x^2 + bx + (\frac{b}{2})^2 - (\frac{b}{2})^2 + c = (x + \frac{b}{2})^2 - (\frac{b}{2})^2 + c$$

   In this case $h = -\frac{b}{2}$ and $k = -(\frac{b}{2})^2 + c$.

   Case II: $a \neq 1$, i.e. complete the square of $ax^2 + bx + c$. First factor a out:

$$ax^2 + bx + c = a(x^2 + \frac{b}{a}x + \frac{c}{a})$$

   Then work with the quadratic in the parenthesis as before:

$$a(x^2 + \frac{b}{a}x + \frac{c}{a}) = a(x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 - (\frac{b}{2a})^2 + \frac{c}{a}) = a(x + \frac{b}{2a})^2 - (\frac{b}{2a})^2 + c$$

   In this case $h = -\frac{b}{2a}$ and $k = -(\frac{b}{2a})^2 + c$.

3. Equations of the form $a(x-h)^2 + k = 0$ can be solved in two ways:

First way (faster): Subtract k from both sides and divide by a. Then take square root of both sides (do not forget the $\pm$) and add h:

$$\begin{aligned} a(x-h)^2 + k &= 0 \\ a(x-h)^2 &= -k \\ (x-h)^2 &= -\frac{k}{a} \\ x-h &= \pm\sqrt{-\frac{k}{a}} \\ x &= \pm\sqrt{-\frac{k}{a}} + h \end{aligned}$$

Second way (usually slower): Expand the square, combine like terms and use the quadratic formula.

1.6 Other Types of Equations

1. If $AB = 0$, then either $A = 0$ or $B = 0$. Use this property to solve quadratic, or higher, equations as follows: Move all terms on the same side. Factor. Set each factor equal to 0.

2. Strategy for solving radical equations, i.e. equations with a square root: Isolate the square root on the one side and everything else on the other side. Square both sides. Solve the resulting equation which does not contain a square root.

   Check if the roots you found satisfy the original equation. If not, reject the roots (*extraneous roots*).

3. Strategy for solving rational equations, i.e. equations where the unknown (x) appears in the denominator: Factor all denominators. Find the least common multiple of all denominators. Multiply all terms in the equation by the least common multiple. Solve the resulting equation which does not contain denominators.

   Check if the roots you found satisfy the original equation. If not, reject the roots (*extraneous roots*).

4. Strategy for solving equations with absolute values: Replace the absolute value with a parenthesis and put $\pm$ in front of the parenthesis. This results in two equations; one with $+$ and one with $-$. Solve them separately.

5. Compound Interest: Assume P is a principal invested for t number of years with interest r. Compounding of interests happens n times a year. Then after t years the amount in the investment is

$$A(t) = P(1 + \frac{r}{n})^{nt}.$$

1.7 Linear Inequalities in One Variable

1. To solve a linear inequality follow the same steps as when solving a linear equality, except you have to switch the direction of the inequality every time you divide, or multiply, by a negative number.

2. If two inequalities are given together, either split the inequalities and solve them separately, or solve all of them together following the same steps as when solving one inequality. Some inequalities can not be solved altogether.

3. Inequalities that contain absolute values. Rewrite the inequalities as follows:

Inequality	Rewrite as
$\|x\| < a$	$-a < x < a$
$\|x\| \leq a$	$-a \leq x \leq a$
$\|x\| > a$	$x > a$ or $x < -a$
$\|x\| \geq a$	$x \geq a$ or $x \leq -a$

4. Interval Notation:

Inequality	Interval Notation
$x < a$	$(-\infty, a)$
$x \leq a$	$(-\infty, a]$
$x > a$	(a, ∞)
$x \geq a$	$[a, \infty)$
$a < x < b$	(a,b)
$a \leq x \leq b$	[a,b]

Bottom line: Use parentheses $(,)$ when the endpoint is <u>not</u> included and brackets $[,]$ when the endpoint is included.

1.8 Other Types of Inequalities

1. Strategy for solving polynomial inequalities: Move all terms on the same side. Factor. Set factors equal to zero and find all roots. Given the roots, split all numbers into all possible intervals. Make a table including all possible intervals and all factors. Take test points. Determine whether each factor is positive or negative for each interval. Determine whether the whole expression is positive or negative for each interval. Find the solution.

2. Strategy for solving rational inequalities: Move all terms on the same side. Combine into a single fraction and simplify. Factor both the numerator and the denominator. Set all factors equal to zero and find all roots. Given the roots, split all numbers into all possible intervals. Make a table including all possible intervals and all factors. Take test points. Determine whether each factor is positive or negative for each interval. Determine

whether the whole expression is positive or negative for each interval. Find the solution.